Design of Asymmetric Filters with Requirements in Two Bands of Finite Extension

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***Abstract*—In this paper, a general design procedure is presented for a class of filters characterized by requirements in two frequency bands, both of finite extension (one passband and one stopband). The filter structure is formed of suitable series resonators coupled through impedance inverters. The resonators also exhibit, in addition to the series resonance in the passband, a parallel resonance in the stopband. The procedure derives a suitable pole–zero distribution for the overall transfer function and determines the parameters of the equivalent resonators in order to obtain a quasi-equiripple response, both in the passband and stopband of the filter. The procedure has been implemented to design filters in rectangular waveguide; a pair of identical filters to be used in a diplexer have been designed through the procedure and fabricated in order to validate the theory.**

***Index Terms*—Circuit synthesis, waveguide filters.**

I. Introduction

It is sometimes required to design filters whose requirements are given in two adjacent frequency ranges, i.e., in a passband and stopband, both of finite extension. This kind of filter, usually defined in the literature as single-sided filters [1], [2], may be advantageously employed, e.g., in the design of diplexers or combining transmission systems; in fact, prescriptions on diplexer performances are very often imposed in only two frequency ranges of comparable extension; using, in this case, a pair of single-sided filters, a smaller number of resonators is required [1], with a consequent reduction of both losses and the overall size of the diplexer [3].

In this paper, an original procedure for the design of single-sided filters is proposed, which is applicable to various filters structures. In particular, an implementation concerning rectangular waveguide filters with narrow or moderate bandwidth is presented. The filters realized are characterized by a quasi-equiripple response both in the passband and stopband. In fact, transmission zeros are suitably introduced in the stopband in order to improve selectivity and minimize the number of required resonators.

An interesting feature of the design procedure, if compared with others found in the literature [1], [2], is that the exact synthesis of an elliptic function prototype is not required (only the low-pass Chebyshev parameters are used in the design). It is then very easy to implement the procedure and the performances obtainable are sufficiently accurate from a practical point-of-view.

The equivalent representation of the filter structure and resonators frequency characteristic are discussed in Section II, where the relevant equations of the design procedure are also derived. Section III describes the implementation of the procedure using waveguide resonators. Finally, computer simulations and experimental performances of a practical test filter are presented and compared in Section IV.

II. Derivation of the Design Equations

The single-sided filters considered here are characterized by the usual equivalent network in Fig. 1, where n-series resonators are cascaded through $n-1$ ideal impedance inverters (normalized external loads are assumed) [4]. All the impedance inverter parameters $K_{i,i+1}$ are equal to one, but for n even, where $K_{n/2,n/2+1}$ is given by $K_{n/2,n/2+1} = \{\varepsilon^2+1\}^{1/2} \pm \varepsilon$ (lower sign for $n = 4, 8, 12, \ldots$, upper sign for $n = 2, 6, 10, \ldots$), where ε is defined by the maximum attenuation in the filter passband $A_m = 10\log(1+\varepsilon^2)$.

The resonators reactance x_k of the kth resonator is assumed to have the following frequency dependence:

$$x_k = A_k \frac{f - f_{pk}}{f - f_{sk}} \tag{1}$$

where f is a suitable frequency variable, which depends on the kind of resonator considered. A_k, f_{pk} and f_{sk} are parameters independent on f, which completely characterize the reactance $x_k(f)$.

Several physical resonators, both lumped and distributed, may be found, which follow, with more or less accuracy, the above frequency characteristic in a limited frequency range. In Section III, a waveguide implementation will be shown.

The filter mask that we assume to be satisfied is shown in Fig. 2. If an equiripple response is imposed in both the passband and stopband, a suitable placement of the resonators poles is required. However, let assume initially that f_{pk} and f_{sk} are not depending on k so that only two frequencies f_p and f_s specify the reactance zero and pole for all the resonators. If a suitable frequency transformation is found, the reactance x'_k of each resonator can be derived from a standard low-pass prototype filter (which is defined once the number n of the resonators and the maximum bandpass attenuation A_m are given) as follows:

$$x'_k = A'_k \frac{f - f_p}{f - f_s} = g_k \omega'. \tag{2}$$

The parameters g_k are the coefficients of the all-poles low-pass prototype [5], here assumed to be of the Chebyshev

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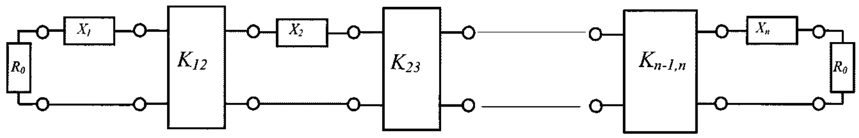

Fig. 1. General configuration of a single-sided filter.

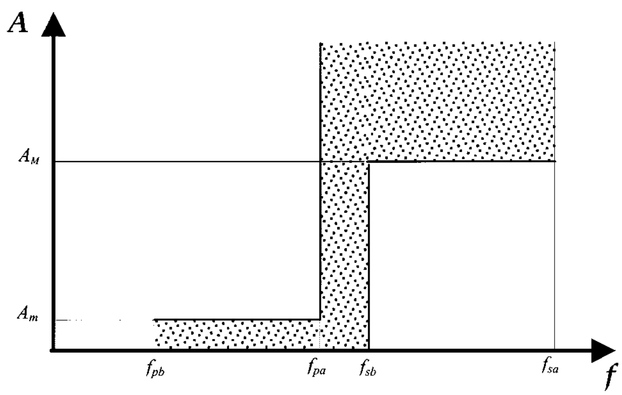

Fig. 2. Attenuation filter mask. $f_{\rm pb}, f_{pa}$ define the passband (maximum attenuation A_m). f_{sa}, $f_{\rm sb}$ define the stopband (minimum attenuation A_M).

kind. A suitable normalized radian frequency ω' satisfying the above relationship may be

$$\omega' = \frac{f_{pa} - f_s}{f_{pa} - f_p}\frac{f - f_p}{f - f_s}. \tag{3}$$

The constants f_p and f_s define the frequency transformation. By requiring the equivalence, in the normalized frequency domain, of the passband extreme frequencies f_{pa} and $f_{\rm pb}$ (Fig. 2), two constraints are posed on (3) as follows:

$$\omega'(f_{pa}) = \omega'(f_{\rm pb}) \quad \omega'(f_{sa}) = \omega'(f_{\rm sb}). \tag{4}$$

From (3) and (4), the following expressions for f_p and f_s can be derived:

$$f_p = \frac{1}{2}\left(S \pm \sqrt{S^2 - 4P}\right) \quad f_s = \frac{1}{2}\left(S \mp \sqrt{S^2 - 4P}\right) \tag{5}$$

where

$$S = \frac{2f_{pa}f_{\rm pb} - f_{sa}f_{\rm sb}}{f_{pa} + f_{\rm pb} - f_{sa} - f_{\rm sb}}$$
$$P = \frac{f_{pa}f_{\rm pb}(f_{sa} + f_{\rm sb}) - f_{sa}f_{\rm sb}(f_{pa} + f_{\rm pb})}{f_{pa} + f_{\rm pb} - f_{sa} - f_{\rm sb}}.$$

The constants A'_k determine the values of the reactances according to (2). Assuming ω' given by (3), they can be expressed as

$$A'_k = g_k \frac{f_{pa} - f_s}{f_{pa} - f_p}. \tag{6}$$

The overall filter, obtained according to the scheme in Fig. 1 once all the reactances x'_k have been determined through the above formulas, will show an equiripple response in the passband (that is between f_{pa} and $f_{\rm pb}$); moreover, n coincident transmission zeros will be obtained at the frequency f_s in the stopband.

We now have to suitably distribute the frequencies f_{sk} between f_{sa} and $f_{\rm sb}$ if an equiripple response is also desired in the stopband. Let note incidentally that the reflection zeros in the passband are given by

$$f_{0k} = \frac{f_s\omega_{0k} - f_p\omega_{sp}}{\omega_{0k} - \omega_{sp}} \tag{7}$$

with

$$\omega_{sp} = \frac{f_{pa} - f_s}{f_{pa} - f_p} \quad \omega_{0k} = \cos\left(\frac{2k-1}{n}\frac{\pi}{2}\right).$$

A possible choice for the transmission zeros f_{sk} is to distribute them in the stopband as the reflection zeros are distributed in the passband. This is obtained by exchanging f_p and f_s, f_{pa} and f_{sa} in (7) as follows:

$$f_{sk} = \frac{f_p\omega_{0k} - f_s\omega_{ps}}{\omega_{0k} - \omega_{ps}} \tag{8}$$

where

$$\omega_{ps} = \frac{f_{sa} - f_s}{f_{sa} - f_p}.$$

Obviously, now having different f_{sk} for each resonator, the frequency transformation (3) will no longer be strictly meaningful; however, new expressions for A'_k and f_{pk} may be suitably derived by requiring two conditions on the passband response of the filter to be satisfied by each resonator.

It has been found that a response very close to the original equiripple is obtained in the passband if the conservation of the reactance x'_k [as given by (2)] is imposed at the frequencies f_{01} and f_{0n} of the first and last reflection zero of the filter (given by (7), with $k = 1$ and $k = n$). The final expressions for A_k and f_{pk} to be used in (1) together with f_{sk} from (8) will then be given by

$$A_k = g_k\omega_{sp}\frac{f_{01} - f_{sk}}{f_{01} - f_{pk}}\frac{f_{01} - f_p}{f_{01} - f_s}$$
$$f_{pk} = \frac{f_{1n}f_{01} - f'_{1n}f_{0n}}{f_{1n} - f'_{1n}} \tag{9}$$

where

$$f'_{1n} = \frac{f_{01} - f_p}{f_{01} - f_s}\frac{f_{0n} - f_s}{f_{0n} - f_p} \quad f_{1n} = \frac{f_{0n} - f_{sk}}{f_{1n} - f_{sk}}.$$

The above equations allow the definition of the single-sided filter in the form of Fig. 1, with a quasi-equiripple response both in the passband and stopband.

As an example of the above synthesis procedure, a three-resonator filter with 10% bandwidth has been designed, and the response is shown in Fig. 3. The following parameters were used:

$$f_{pa} = 1,\ f_{\rm pb} = 0.9,\ f_{sa} = 1.05,\ f_{\rm sb} = 1.15,\ A_m = 0.1 \text{ dB}.$$

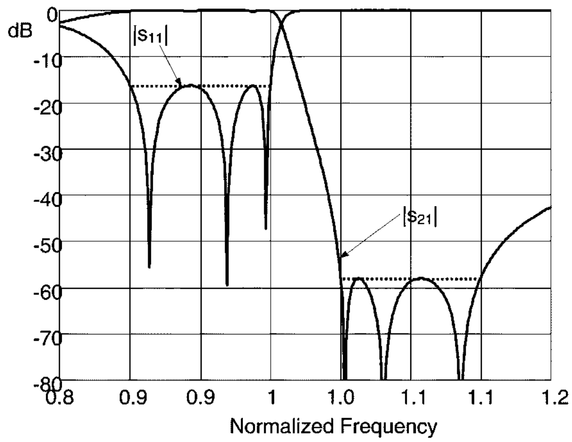

Fig. 3. Response of a three-resonator test filter with: $A_k = [-3.8895,\ -3.0039,\ -2.1028]$, $f_{pk} = [0.9649,\ 0.9691,\ 0.9730]$, and $f_{sk} = [1.1362,\ 1.0809,\ 1.0531]$.

It is evident from Fig. 3 that the filter response is practically equiripple both in the passband and stopband. Note that the maximum return loss in the passband is 14.43 dB (corresponding to $A_m = 0.1$ dB), while the minimum attenuation in stopband is 58.2 dB (it is determined by the number of resonators and by the separation between the two bands).

III. Waveguide Filters Realization

The filter design procedure previously developed can be applied to design rectangular waveguide filters that may be used, for instance, in diplexers applications. The main problem here is concerned with finding a configuration for the series resonators in rectangular waveguide that presents, with a sufficient accuracy, a reactance versus frequency in the form of (1), at least in a restricted frequency range. The impedance inverters are realized, as usual, by quarter-wavelength waveguide sections.

Let us first define a suitable frequency variable to be used in (1) for the present case

$$F_g = \sqrt{f^2 - f_c^2}. \tag{10}$$

F_g is the waveguide frequency usually considered in waveguide filters design. f_c is the cutoff frequency of the dominant mode TE_{10} and f is the actual frequency (denormalized). Equation (10) is used to transform the frequency specifications of the filter, given in the f domain, into new specifications in the F_g domain, where the design procedure will be applied.

Equation (1) then becomes

$$x_k = A_k \frac{F_g - F_{g,pk}}{F_g - F_{g,sk}} \tag{11}$$

where A_k, $F_{g,pk}$, and $F_{g,sk}$ are the three parameters that characterize the waveguide resonator in the F_g domain.

The equivalent circuit for the proposed waveguide resonators is depicted in Fig. 4. y_{ck} represents the characteristic admittance of the series-connected short-circuited stub (normalized to the main waveguide). x_{pk} and b_{sk} are frequency-independent reactive elements.

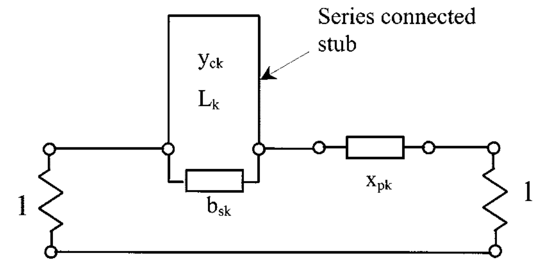

Fig. 4. Equivalent circuit of the waveguide resonator.

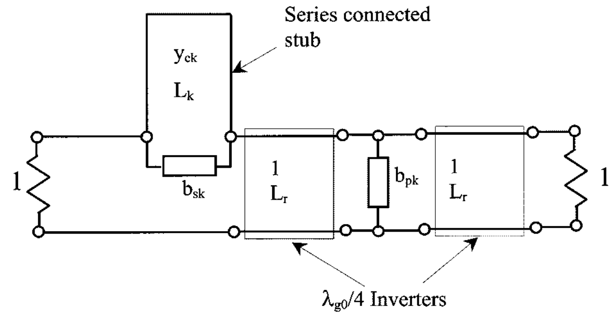

Fig. 5. Equivalent circuit of the waveguide resonator with the reactance x_{pk} transformed into a shunt susceptance b_{pk}.

The reactance of this resonator can be put in the form of (11) if b_{sk} and x_{pk} are given as follows:

$$b_{sk} = \frac{y_{ck}}{\tan(\phi_{sk})} \quad x_{pk} = \frac{\tan(\phi_{pk})\tan(\phi_{sk})}{y_{ck}\left[\tan(\phi_{pk}) - \tan(\phi_{sk})\right]} \tag{12}$$

with

$$\begin{aligned}\phi_{pk} &= \beta_{pk} L_k = \left(\frac{2\pi F_{g,pk}}{c}\right) L_k\\ \phi_{sk} &= \beta_{sk} L_k = \left(\frac{2\pi F_{g,sk}}{c}\right) L_k.\end{aligned}$$

L_k is the stub length and c is the light velocity.

It then has for x_k

$$x_k = \frac{\tan^2(\phi_{sk})}{y_{ck}\left[\tan(\phi_{pk}) - \tan(\phi_{sk})\right]} \frac{\tan(\phi_k) - \tan(\phi_{pk})}{\left[\tan(\phi_k) - \tan(\phi_{sk})\right]} \tag{13}$$

where $\phi_k = (2\pi F_g/c)L_k$.

If ϕ_k, ϕ_{pk}, and ϕ_{sk} are close to π (that is for a narrow or moderate frequency range), then $\tan(\phi) \cong \pi - \phi$ and x_k becomes

$$x_k = \frac{\tan^2(\phi_{sk})}{y_{ck}\left[\tan(\phi_{pk}) - \tan(\phi_{sk})\right]} \left[\frac{F_g - F_{g,pk}}{F_g - F_{g,sk}}\right] \tag{14}$$

which just reproduces (11) if L_k is derived from the following equation:

$$\frac{\tan^2(\phi_{sk})}{y_{ck}\left[\tan(\phi_{pk}) - \tan(\phi_{sk})\right]} - A_k = 0. \tag{15}$$

TABLE I
RESONATORS PARAMETERS OBTAINED FROM DESIGN (THE VALUES BETWEEN PARENTHESIS WERE OBTAINED AFTER OPTIMIZATION). THE LENGTH OF THE TWO WAVEGUIDE SECTIONS BETWEEN THE RESONATORS ARE 14.17 AND 14.15 mm, RESPECTIVELY

	L_s (mm)	b_{sk}	L_r (mm)	b_{pk}
Resonator 1	11.863 *(11.866)*	-2.31 *(-2.34)*	7.0621 *(7.045)*	-2.026 *(-1.889)*
Resonator 2	12.627 *(12.620)*	-3.50 *(-3.52)*	7.0455 *(7.089)*	-1.819 *(-1.693)*
Resonator 3	13.295 *(13.298)*	-6.939 *(-6.98)*	7.0325 *(7.062)*	-0.847 *(-0.8)*

Once L_k is determined, b_{sk} and x_{pk} can also be computed by means of (12) and the resonator is completely defined in terms of A_k, $F_{g,pk}$, and $F_{g,sk}$. Note that the parameter y_{ck} can be arbitrarily assigned (y_{ck} determines the dimensions of the waveguide sections that implement the series-connected stubs). This degree of freedom may be used to obtain a value of L_k from (15) close to a half-wavelength at the center frequency of the filter passband in order to meet the assumptions on the electrical lengths.

It can be observed that the proposed configuration of the waveguide resonators is not suitable for a practical realization; however, the series reactances x_{pk} can be conveniently transformed into equivalent shunt-connected susceptances b_{pk} through two quarter-wavelength inverters (Fig. 5). As can be easily deduced, the numerical values of b_{pk} are equal to x_{pk}.

From a practical point-of-view, the series stubs and parallel susceptances b_{sk} are realized by means of cavities coupled to the E-plane of the main rectangular waveguide with a suitable iris, depending on the sign of b_{sk}. The other susceptances b_{pk} are implemented as inductive or capacitive windows in the main waveguide cross section. Note that once the resonators are cascaded with the filter impedance inverters, the waveguide sections on the right-hand side of each b_{pk} and the quarter-wavelength sections of the filter inverters combine into half-wavelength waveguide sections.

In conclusion, the design procedure for waveguide single-sided filters can be summarized as follows.

1) The characteristic parameters A_k, $F_{g,pk}$, and $F_{g,sk}$ of each resonator are computed with (8) and (9), given the following filter specifications: passband and stopband extreme frequencies, number of resonators, maximum passband attenuation [the frequencies have to be transformed into the F_g domain through (10)].
2) The susceptances b_{pk}, b_{sk}, and the length L_k of the stubs in the equivalent circuit of the waveguide resonators are computed through (12)–(15); the values of y_{ck} are freely selected, according to the previous observations.
3) Suitable waveguide discontinuities are selected to realize the susceptances b_{pk} and b_{sk}; their physical dimensions are computed using the available formulas and charts from the literature [6].
4) The lengths L_r of waveguide sections inside the resonators are computed at the frequencies F_{pk}; the lengths of the filter inverters are computed at the center of the filter passband.
5) Suitable tuning elements, such as screws, should be provided near both the junction of the cavity stubs, where the discontinuity effects have to be compensated, and in the middle of the waveguide sections to allow small variations of the equivalent length and impedance.

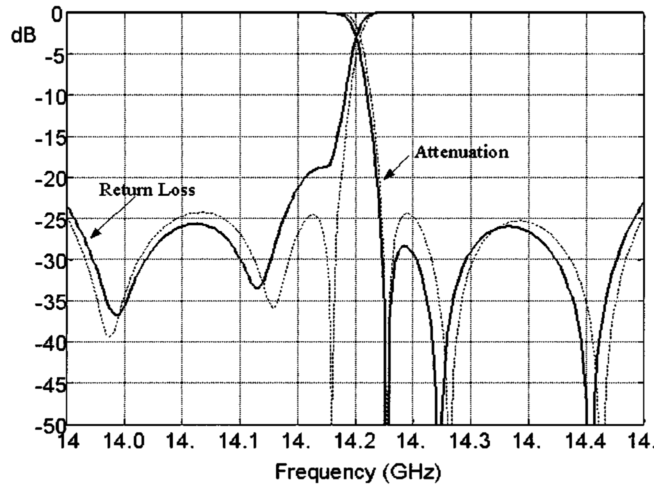

Fig. 6. Simulated frequency response of the test filter. Continuous curves refer to the design results. Dashed curves have been obtained after frequency optimization.

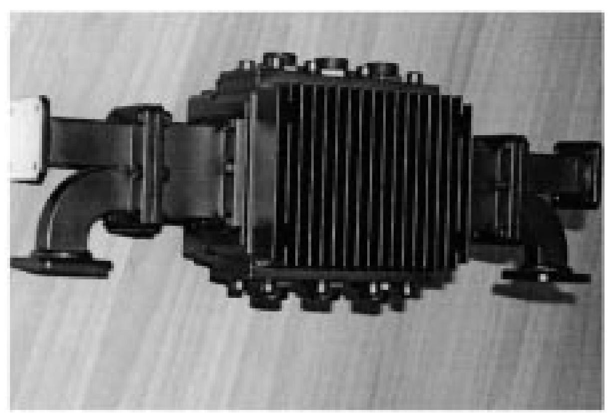

Fig. 7. Overall diplexer. It includes two filters and two 90° hybrids.

IV. DESIGN EXAMPLE

The design procedure proposed in this paper has been employed to design a pair of identical single-sided filters to be used in a diplexer for a satellite transmission system in the Ku-band.

The filter electrical requirements are as follows:

passband	14.000 –14.235 GHz;
stopband	14.273 –14.500 GHz;
minimum return loss in passband	25 dB;
minimum attenuation in stopband	25 dB.

To satisfy the above specifications, at least three resonators are necessary. The parameters L_k, L_r, b_{pk}, and b_{sk} obtained by applying the design procedure as outlined in the previous sections are reported in Table I.

The filter has been simulated using waveguide sections (WR62) and frequency-invariant susceptances; the computed

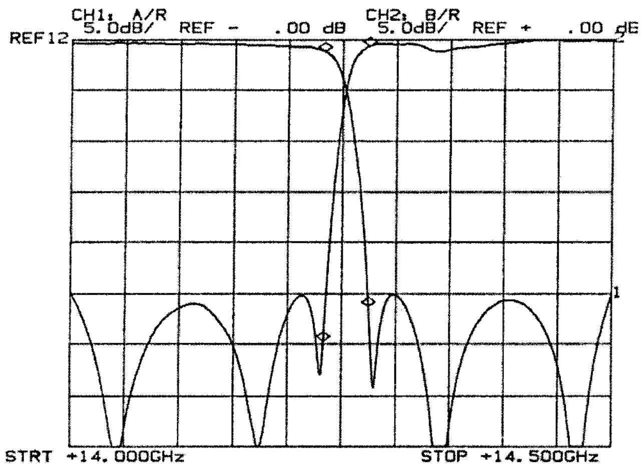

Fig. 8. Measured response of the test filter.

response thus obtained (attenuation and return loss), shown in Fig. 6, may be considered satisfactory even if the filter specifications are not completely met, especially in the passband. These discrepancies are, in fact, sufficiently small to be easily eliminated during the filter experimental tuning. In any case, an optimization procedure has been applied to the initial design: the final values of the parameters are also reported in Table I (note the very small variation compared with their initial values), while the optimized filter response is also shown in Fig. 6 (dashed curves).

Two samples of the designed filter have been fabricated, together with two 90° hybrids in order to realize the diplexer as a directional filter [5]. A picture of the very compact overall structure is shown in Fig. 7, while the measured response of each filter is reported in Fig. 8 (note the very good agreement with the simulated response).

V. CONCLUSIONS

A new design procedure for a class of waveguide filters has been presented, which allows to control the filter response in two adjacent frequency bands; the response is equiripple (with a good approximation) both in the passband and stopband. A structure for the waveguide resonators has been proposed and the design equations for the relevant elements have been derived. Simulations and experimental results from filters actually realized have confirmed the effectiveness of the design procedure.

ACKNOWLEDGMENT

The authors wish to dedicate this paper to the memory of E. Cavalieri d'Oro, who, many years ago, experimented with waveguide single-sided filters such as those described in this paper.

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